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Capitalism beyond harmonious equilibrium: mathematics as if human agency mattered

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Abstract. Narrating a world of flux entails moving away from equilibrium-oriented thinking toward considerations of emergence, uncertain futures, and unintended consequences. This is not the exclusive domain of the qualitative theory construction and analysis that has dominated such thinking in sociospatial theory: it is also involved in mathematical theory construction. It requires a relational approach to mathematical theory, however, that moves beyond unidirectional claims of cause and effect, avoids deterministic and teleological thinking, and recognizes the incompleteness and openness of any such theoretical construction. These arguments are explored through an example that employs mathematical techniques often associated with complexity theory to examine unevenly shifting economic landscapes where the best guesses of capitalist entrepreneurs are interrelated with the emergent multiregional economy in which capitalists participate. This highlights the unexpectedly heightened dynamical importance of regions in a globally connected world; how cherished theoretical principles become renegotiated, as relationality leads to emergence; and that there is space for human agency, through modeling praxis appropriate to 'incomplete systems'. We open modest cracks in the supposed wall between quantitative and qualitative approaches, oriented toward a methodological reinterpretation of what employing mathematical arguments could mean within larger, postpositivist theoretical projects in critical human geography.

1 Introduction: mathematics and contemporary sociospatial theory

> "A method of the rhizome type, on the contrary, can analyze language only by decentering it onto other dimensions and other registers. A language is never closed upon itself, except as a function of impotence."
>
> Deleuze and Guattari, 1987 *A Thousand Plateaus* (page 8)

'Critical' human geography has become a highly multifaceted affair. In their editorial introducing *ACME*, the e-journal of critical geography, Moss et al define it thus:

> "for example, anarchist, anti-racist, environmentalist, feminist, Marxist, postcolonial, poststructuralist, queer, situationist, and socialist. By critical thinking and radical analysis we mean that the work is part of the praxis of social and political change aimed at challenging, dismantling, and transforming prevalent relations, systems, and structures of capitalist exploitation, oppression, imperialism, neo-liberalism, national aggression, and environmental destruction" (2001, page 3).

Notwithstanding this diversity of subject matter, postpositivist philosophical dispositions in geography and related social sciences have tended to be less inclusive of methodological diversity. Specifically, and recently prominently discussed, there has been skepticism about, and lack of attention to, quantitative approaches—ranging from the use of mathematics as a language of theory, through to quantitative models, statistics (including qualitative statistics), and geographic information systems (GIS). Such exclusions are valid for particular research programs, but it is problematic to claim that 'quantitative' approaches are inappropriate in critical geography and sociospatial theory tout court because they are implicitly positivist and oriented toward reproducing, rather than critically assessing, the increasingly capitalist status quo (Sheppard, 2001). These issues continue to be intensively debated (Hubbard and Kitchin, 2007; Johnston, 2006).

In this paper we argue that the use of some mathematical languages to theorize the socionatural world is by no means inconsistent with the emergent sensibilities of 'critical' geography or with its recent 'relational' turn—one that has brought attention to the relevance of the thinking of such scholars as Donna Haraway, Judith Butler, Bruno Latour, and Gilles Deleuze and Felix Guattari for sociospatial theory. In contrast with the assumptions underlying conventional usages of mathematical models in the social sciences, the works of many current theorists narrate the socionatural world as continually shifting, spatiotemporality as constructed, reductionist and essentialist approaches to explanation as undesirable, and relational social ontology as important. We demonstrate that certain mathematical discourses and practices can be used to advance a relational social ontology, which conceptualizes entities in terms of the relations though which they come to take on particular qualities in particular contexts, rather than in terms of their possession of a set of essential properties.

The emergence of 'complexity theory', including new mathematical and modeling approaches, has drawn attention to ways in which postmodernism, poststructuralism, Deleuzian thinking, and nonteleological dialectics can resonate with mathematics. Yet such discussions have still largely occurred outside geography (Cilliers, 1998; DeLanda, 2006; King, 2001; Rosser Jr, 2000; Urry, 2003). Despite growing interest within critical geography in the intersection between complexity theory and social theory (Bonta and Protevi, 2004), few have attempted to bridge the apparent divide between them when each is written in its 'native tongue', one quantitative and the other qualitative. Some, such as Plummer and Sheppard, have begun a dialogue between these, arguing that the assemblage of mathematical theorizing associated with 'complexity theory,' notably those of nonlinear out-of-equilibrium dynamics, offers theories about the world with many of the relational properties that geographers conventionally associate with qualitative languages of theory construction (Plummer and Sheppard, 2006; 2007).

For example, sociospatial theorists have critiqued what have been called 'Newtonian conceptions' of space/time, in which space/time are exogenously given coordinates in Euclidean (or Hilbert) space, replacing this with a relational approach that stresses space/time as emergent and constructed. However, the latter can also be found in mathematical models of complex dynamical systems. Prigogine (1996) notes that, within this domain of mathematics, spatiality and temporality are emergent properties constituted through the persistent nonlocal interactions connecting system elements. In such systems broader-scale and local-scale phenomena are mutually constitutive. It is neither the case that larger scales necessarily dominate local events, nor that aggregate outcomes are necessarily the consequence of individual actions. Scale—both the scales that come into existence and their relative import in shaping spatial dynamics—can be constituted by, rather than merely being an exogenous parameter of, the system. This is reminiscent of Soja's sociospatial dialectic, Harvey's dialectical conception of space/time, and Massey's version of relational space (Massey, 2005; Sheppard, 2006; Soja, 1968). In the example developed below, scale is not explicitly represented, but dialectical relations between individual actions and regional or aggregate dynamics nevertheless emerge.

To take a second example, the desire to use mathematics to make general deductive claims about the world that may be subjected to empirical testing (popularized in the logical positivism of the Vienna Circle) has pushed social scientists away from relational thinking. Thus in mainstream economics and in geographical economics, extreme assumptions about human behavior and methodological individualism ('microfoundations') are commonly appealed to, on the grounds that they are not only philosophically desirable (to some), but also necessary (given the current limitations of mathematical theory) in order to deduce general theorems about economic equilibrium (Krugman, 1995). The remarkable strides made by computational approaches in recent decades have vitiated many such arguments, however, enabling deductive theorem-proof approaches to be supplemented increasingly by a wider realm of theoretical perspectives capable of being explored by computationally intensive simulations and scenario building.

Of course, it is important to consider whether the methods chosen are appropriate to the task. As Martin and Sunley note, loose metaphors drawn between the type of problem addressed by one of the many 'complexity' approaches developed in the physical or biological sciences and a human-geographical phenomenon may not be suitable (Martin and Sunley, 2007). From this, they conclude that geographers should employ an approach less centered on the particular mathematical models of complexity; they should rather ensure that our theories ontologically embrace empirical economic-geographical landscapes as possessing a set of properties which they associate with complexity, such as being open, highly interconnected, self-organizing, emergent, and adaptive (page 596). While many existing *particular* 'complexity' approaches are inappropriate for import into human geography, in our view, there is still much to gain from engaging with the modeling processes which were critical in generating the abstract principles of complexity that Martin and Sunley identify. When a mathematical/computational model is derived from assumptions that are theoretically justified in our own field (as we attempt below), it is more likely that the metaphors and results are ontologically appropriate, and it is still possible to take advantage of the other benefits to modeling approaches for which we argue here.

To avoid any misapprehension of our intentions, it is worth clarifying our position on mathematics and 'critical' geography before proceeding. We do not seek to substitute mathematical reasoning for the powerful verbal theoretical arguments that have come to characterize critical geography. We view mathematics as a particular language, perhaps with a variety of dialects and genres, but also with its own well-defined internal rules of grammar and communication; with incomplete truth claims (Gödel, 1931); and with abilities to provide new insights into qualitative arguments. The use of mathematics cannot provide a license for us to evade responsibility for the particular philosophical and political implications of our theories. Quantitative reasoning too often becomes the scapegoat in criticisms of mainstream social science, when the framework within which such quantitative approaches are embedded is, in fact, often more to blame.

The relatively strict grammatical and formal constraints of the language of mathematics are not merely restrictive, but provide a framework in which creative theorizing may flourish. For all its limitations, the use of mathematical reasoning does often force investigators to clarify the exact nature of the assumptions they wish to make about society, and enables them to trace their unexpected consequences. All too often, when we advance a principle, we consider it in isolation from the many complicated feedbacks that might appear—or even come to dominate that principle—if we had a way to simultaneously consider how that principle might be indirectly affected by our other principles, even those that do not appear to be directly related. In isolation, such principles tend to become reified as determining general outcomes, whereas, in fact, they may be no more than tendencies, whose influence in a relational world may be partial at best. Within a dynamic environment, 'optimizing' should always be seen to be radically different from having been 'optimized'. For example, assuming that capitalists seek to maximize their profits does not mean that their actions result in profit maximization, because actions are always undertaken in the presence of many other simultaneous social processes. The mathematics of dynamical systems, often studied through computation, make it possible to incorporate such relationalities and explore their consequences under carefully specified conditions.

When we use appropriate forms of mathematics and computation to place several sociospatial assumptions written in mathematical language into interaction with each other, we gain the ability to begin to move beyond simple dichotomies. Unidirectional causality, or cause–effect reasoning, is one such dichotomy. In a relational approach to mathematics, thousands of causal chains and processes are in play, with any one event overdetermined by the contingent consequences of many possible influences. In the process, any particular unidirectional causal relation potentially becomes dominated by the ways in which everything is potentially, directly or indirectly, related to everything else over time. We refer to this as a shift from unidirectional to emergent causality.

Some have begun to pose the question of what it would mean to move beyond conventional notions of causality. Thus Althusser, pondering Spinoza in the final chapter of his extended essay "The Object of *Capital*" (in Althusser and Balibar, 1997 [1968]), wondered how one might move beyond what he saw as the two dominant ways of thinking causality: through a unidirectional cause and effect or through a concept of systemic essences expressing themselves. He rejects these by imagining a more nuanced formulation associated with a notion of overdetermination, based on complex 'structures' that are not governed by systemic totalities but whose interrelations are critical, for which he poses a question:

> "By means of what concept, or what set of concepts, is it possible to think the determination of the elements of a structure, and the structural relations between those elements, and all the effects of those relations, by the effectivity of that structure?" (page 186).

Althusser poses the critical question of how we can move beyond simple reductionisms, but perhaps neither he nor his critics had the language to go much further in working out the consequences of this new causality. (For a discussion of overdetermination in Harvey's theory of capitalism, see Resnick and Wolff, 2004.)

We suggest that mathematics offers a language in which such emergent causality might become expressed and explored. In the example developed below, we illustrate how a complexity modeling approach can lead to insights into the mutual transformation over time of various social processes as part and parcel of the dynamics of the capitalist space economy. However, in the spirit of a relational approach, and of reflexively acknowledging, critically interrogating, and altering the limits that accompany any mathematics, we seek to be cognizant of several limitations that we view as being interrelated in some mathematical approaches: determinism and teleology (whereby outcomes are already predetermined by the mathematics of the system), as well as closure (the position that a mathematical model captures within it all relevant features of reality).

With respect to determinism, the study of nonlinear dynamics and chaos has, of course, popularized the possibility that we cannot predict the future deterministically. Chaos and complexity have emerged as discourses offering constructive approaches by which we can move beyond the false dichotomy between total certainty and complete inability to make meaningful comparisons of possibilities. Though we view such revelations as critical and underappreciated, here we address a different, perhaps more metatheoretical, issue. Certain difficulties arise less from the abstract features of a type of mathematics and more from the ways in which we narrate the use of our mathematics.

The relationships between mathematical models and qualitative narratives are quite complicated, in theory and in practice. Others have explored in greater depth how models might be narrated (Guhathakurta, 2002), how narratives have been critical in building and interpreting models (Morgan, 2002), and how subjective narratives can complement simulation models while addressing subtle issues of meaning and motivation (Uprichard and Byrne, 2006). In exploring the narration of a model, we are also concerned with questioning whether we modelers have too often partaken of certain practices that lead us to narrate all mathematics (even those which use the language of probability) deterministically and teleologically. When a complex set of mathematical equations expressing social processes are put in motion with respect to one another, our current cultural impulse is to try to 'solve' the system definitively—to figure out the various paths the system would take, depending on the boundary/initial conditions and the values of various so-called parameters. 'Solving' the equations (analytically or numerically) can be a significant prerequisite to many forms of analysis. These are technical steps which we necessarily endorse. After solutions are obtained, however, a fraught moment often arrives when one interprets the solutions as inviolate, as determined by the system of equations and determining of the future of the system being theorized.

This act of positing an unproblematic connection between mathematical solution and that which is being theorized usually requires an implicit assumption that the system is 'closed'—that the equations adequately express all of the various factors and processes relevant to society. Of course, no social system can be 'closed'. We thus suggest paths along which a conversation might arise about mathematical practices that embrace the 'incompleteness' of their systems and are potentially able to integrate intricately with qualitative theory. Embracing this perspective can lead us to develop methods which incorporate both epistemological modesty and human agency into mathematical narratives whch are nonteleological and nondeterministic.

In order to explicate these general claims, in the remainder of the paper we explore a model with the above arguments in mind—one which explores the spatial dynamics of capitalism. This model is in the tradition of classical political economy, informed by Marx's dialectical theorization of how commodity production entails interlinked circuits of money, commodities, and the means of production (Marx 1967 [1867]; 1983 [1857–58]). In section 2, we begin by reviewing debates about the dialectical relation between capitalists' actions and the aggregate dynamics of capitalism, as well as about the possibility of equilibrium. We then develop a model of capitalists' actions and reactions with respect to commodity production and the circulation of capital, developing a dynamical model that can operate far-from-equilibrium and which, when read properly, avoids teleology and determinism. In section 3 we use computational techniques to examine the polyvalent dynamics of this system and show that, whereas various dynamical equilibrium patterns often emerge in the long run which direct or bound accumulation dynamics, collapse and polarization is also possible (under different conditions from those in geographical economics). Short-run dynamics, governed by unidirectional causality, can become overwhelmed by longer run relational processes governed by emergent causality. Yet the transient paths traveled before any type of dynamical equilibrium could be reached remain especially significant when teleology is avoided, and incompleteness acknowledged, because the 'blooming, buzzing confusion' too often forgotten in the stylized, sanitized 'reality' of equilibrium economics may be better approximated by stringing together short-run scenarios, as part of the creation of a methodological opening to the consideration of mathematical systems in their 'incompleteness'. Section 4 concludes the paper.

2 The circulation of capital and the spatial dynamics of accumulation

Marx advanced the claim that capitalism is inherently unstable, characterized by a dynamic and dialectical tension between a variety of social processes. Harvey (1982) interrogated the geography of this claim by spatializing Marx's theory of uneven geographical development, arguing that the spatiality of capitalism increases its complexity (see also Castree, 1999; Sheppard and Barnes, 1990; Storper and Walker, 1989; Swyngedouw, 1992; Webber and Rigby, 1996). Perhaps in part reflecting Harvey's own skepticism of quantitative geography after 1970, few of the scholars developing such arguments have attempted to express such arguments mathematically. Those who have done so have confirmed Harvey's argument that the spatiality of capitalism, itself a product of capitalist production and exchange, reinforces its contradictory and unstable nature (eg Sheppard and Barnes, 1990; Sheppard et al, 1992). Yet their examination did not extend to the nature of the pervasive out-of-equilibrium spatial dynamics of capitalism that may be regarded as a corollary of this argument. Instead, there was a tendency to confine analysis to the nature and (in)stability of dynamic market equilibria; the path followed by a growing, self-reproducing capitalist society (Marx's scheme of expanded reproduction).

The approach taken here differs from such progenitors by avoiding a focus on equilibrium. Instead, we let any equilibria that may emerge do so as organic 'unintended' consequences of social processes. In a disequilibrium framework, the coevolution of dialectically interrelated microperspectives and macroperspectives can be stated in terms of tendencies, instead of a priori laws or simplified deterministic relationships. By contrast, theorizing that assumes the possibility and the centrality of equilibrium flirts with the traps of teleological and deterministic reasoning, reproducing and reifying equilibria, and obscuring the potential synergies between qualitative and quantitative approaches to critical geography. We also avoid the methodological individualism permeating some mathematical models of Marx—those associated with rational choice Marxism (Roemer, 1981)—examining instead the dialectical relation between the actions of individual capitalists and macroeconomic conditions. Finally, we keep in mind that any such model is highly incomplete, since much is left out, which further calls into the question any notion of making deterministic predictions about the spatial dynamics of capitalism.

2.1 The actions of capitalists and the spatial dynamics of accumulation

We draw inspiration for our discussion of the interrelations of the actions of particular capitalists with the aggregate dynamics of capitalism from the problematization and initial analyses found in Marx's *Das Kapital*. Each capitalist is differently situated within space–time, seeking to enhance the profitability of his or her investments under the pressure of (spatial) competition. In abstract discussions of the circuits of capital, the actions of capitalists are conceived as a continually intertwining stream, dialectically articulated with the macrodynamics of capital. Marx expends much effort in volume 2 of *Das Kapital* exploring the challenges and incoherencies that result from the manifold ways in which these actions may or may not articulate with one another (Marx, 1885 [1972]).

Each individual process of production takes time and stretches across space. Capitalists take risks: advancing money capital to purchase labor and nonhuman inputs; using these to produce a commodity for sale, drawing on prevailing production methods; sending that commodity to market; and waiting for revenues to be generated from sales which then return (hopefully) as accumulated money capital (realized profits) for investment in new production. The time that passes is referred to as the production period—the time during which the capitalist waits to see whether the capital advanced does indeed generate the anticipated profits. Much can go awry between the beginning and end of any firm's production period that may undermine the profitability of a capitalist's initial investment (measured here as the rate of profit per unit of money capital advanced, per year). The capitalist is uncertain about the prices and availability of inputs; faces the possibility that production does not go according to plan; and cannot be entirely sure how much of the commodity will be sold, where, and at what prices. Rational expectations are impossible.

In an attempt to stylize the aggregate picture for the purposes of analysis in volume 3 of *Das Kapital*, Marx collapses this stream of interpenetrated, spatiotemporally overlapping circuits of capital into a common production period; and aggregates the many kinds of commodities produced into two: a wage and a capital good (Marx, 1896 [1972]). Developing a scheme of expanded reproduction, he seeks to determine the conditions under which it is possible for capitalism to reproduce itself ad infinitum, in such a way that profit rates are equalized across the sectors, and the quantities produced within each sector are all sold. [We leave aside questions of labor value (cf Sheppard, 2004).] He also seeks to examine how local actions, at particular moments in time and in particular places, relate to longer term and broader scale aspects of capitalism. It is in such arguments that we find Marx exploring how neither the 'micro' nor the 'macro' perspective could be determinant of the other, but are dialectically related—mutually overdetermined, with their coevolution describable in terms of tendencies, not laws.

Radical economists using mathematics to explore the general principles that can be abstracted from Marx's numerical examples have also focused on characterizing this scheme of extended reproduction as a dynamical 'long-run' equilibrium—in effect constructing Marxian general equilibrium models (closed, like all such models, to other influences). This describes a trajectory for capitalism along which output would hypothetically exactly match subsequent demand, competition would enable all capitalists to realize the same average profit rate, and the economy would grow ad infinitum (eg Morishima, 1973; Pasinetti, 1981; Roemer, 1981; Walsh and Gram, 1980). This 'golden growth path' can be precisely determined, in terms of a unique set of relative production prices, and relative production quantities, which equalize the rate of profit (and determine a constant rate of growth) for any given set of production technologies, labor relations, and consumption norms. Note that, along such a dynamic equilibrium path, time is reversible, spatiality is fixed, and history does not play any significant role. The golden path as defined in this way represents conditions under which Say's Law, much criticized by Marx and Keynes, actually holds: that is, supply technically creates its own demand, and markets clear.

In seeking to understand the dialectical relations between microprocesses and macroprocesses under capitalism, there has been intense debate about whether the golden growth path is consistent with the profit-seeking actions of individual capitalists. If it is, then it becomes plausible to imagine that the economy can be observed in this state. There have been attempts to model the 'price–quantity' dynamics (for an aspatial economy) that result from capitalists' actions, starting from the presumption that the economy is far from equilibrium, in order to ask whether the actions of capitalists will drive it towards the golden growth path. Even if technologies, wages, sociopolitical power, and the socionatural context are unchanging, can the economy adjust in relative harmony, moving coherently toward the golden growth path? If so, can we say anything about the likelihood of convergence? And what would be the implications, in more narrowly economic terms or in broader theoretical context, if this moment of the larger socionatural dialectic were always not in equilibrium?

While convergence to the golden growth path has been demonstrated under certain conditions (eg Duménil and Lévy, 1993), this is predicated on the rather strong presumption that capitalists are aware of—and thus know to what degree they may be deviating from—the conditions defining the golden path, and that they respond accordingly. In short, an assumed awareness of the long-term/broader context is often theorized, thereby shaping short-term/local behavior. Treating the golden growth path as a known macroproperty of capitalism may have the unintended (teleological) effect of embedding greater tendencies toward convergence within the model. While a useful step forward, such models are limited in their dynamic plausibility and potential theoretical insight by the explicit assumption that the golden growth path exists and that individuals are aware of it.

By contrast, we make no such presumptions about the causal efficacy of processes which, taken in artificial abstraction, would reinforce a tendency toward—and then perhaps likelihood of—equilibrium. Capitalists in particular sectors and regions simply respond to limited commonplace understandings about current economic conditions in that sector/region, leaving open the question of whether the golden growth path is an emergent feature. As we will show, 'families' of golden growth paths do emerge. However, taking a more dynamic approach to the issue forces a reconsideration of how causality within complicated systems should be analyzed, with short-term dynamics clearly being dialectically related with long-term dynamics—each driven by different considerations. Perhaps most interesting, philosophically, are the ways in which human agency might then be naturally integrated within mathematical approaches which otherwise (such as in more equilibrium-oriented models) might appear unrealistically deterministic or teleological.

2.2 Description of the model

In this section we present the model developed following the principles described above. The mathematical details are given in an online appendix A (see http://dx.doi.org/10.1068/a411ap). Our model does not build on all of the diverse modern complexity approaches, but we choose a minimal set of more classic nonlinear dynamical methods to explore the plausibility of more abstract ontological claims. In exploring links between a mathematical model of the capitalist space economy and contemporary sociospatial theory, we consider here a highly simplified theoretical capitalist space economy. Imagine a space economy made up of two regions, each of which can engage in the production of two commodities: a wage and a capital commodity. In reality, production methods and labor relations are unequal and are in flux, the division of the surplus among workers and capitalists is contested, industrial sectors are internally heterogeneous, individual regions are spatially heterogeneous with distinct internal conditions and locational advantages, and production periods are heterogeneous and of variable length as capitalists pursue strategies to accelerate the turnover of capital. Here, however, we force this stream of actions into the straightjacket of an iterative sequence of production periods (indexed by time) that are of equal length across each sector and region (although this can be generalized to production periods of unequal length). Thus the model to be developed here is best thought of as an attempt to explore the complexities and instabilities of spatial intercapitalist competition that emerge even in this highly stylized situation.

Consider the simplified example of two regions (A and B) and two commodities (wage and capital goods). For any production period, t, capitalists in each sector/region base their actions on heuristics oriented toward the pursuit of profit, in response to what they experienced in the previous time period. Careful accounting for quantities in the model is associated with an attention to necessary interdependence (and subsequently, coupled dynamics) among sectors and regions constrained by (short-term invariant) production technologies. The many circuits of money capital and physical goods are complicated and interwoven, as reflected by the equations in appendix A, meaning that any linear sequence employed in the discussion of equations, including the one below, is always a rhetorical compromise.

At the beginning of any production period, capitalists have access to investment capital, K (in our approximation, this excludes fixed capital). We assume that capitalists will invest all of K in expanded production (termed 'planned supply') in their sector/region during the production period, to be completed and brought to the market in the next period. If shortages exist in the real supply of the various required inputs (as produced in the various sectors/regions during the previous time period), this will constrain capitalists from placing all of their resources into production, and actual output ('real supply') will be less than planned supply.

For each sector/region, the proportion of each required input to be purchased from each of the two regions, in order to produce the commodity to be supplied, depends both on the relative delivered 'bid' price and on the size of that sector in each region, and determines desired/'planned' demand (that is, total sum of an input desired to be purchased by various sectors during this time period). As implied above, realized demand may be less than planned demand, pushing real supply down below planned supply (and preventing the firm from investing all its capital in production). On the other hand, a lack of sufficient demand for the realized supply of a sector results in less income and lower profitability for the sector in question.

The amounts of inputs which sectors are able to purchase in pursuing their production plans determine the total expenditures, E, in each sector/region. If expenditures are less than the investment capital available, then the sector/region accumulates unspent investment capital, K^x. Revenues, y, are calculated for each sector/region, based on realized demand and current prices. Total capital accumulated by the end of the time period, Y, is the sum of revenues and excess (unspent) capital. Profit rates are calculated for each sector/region, based on revenues at the end of the production period relative to investment capital available at the start of the period. Any differences in profit rates between sectors and regions will result in capital flows (the reallocation of the total capital accumulated) from less profitable to more profitable sectors/regions. This determines the investment capital available in each sector/region for the next time period.

The pricing sphere operates as follows. The price, p, at which capitalists believe they will be able to sell their output in the market, is adjusted from the price of the previous period, taking into account supply/demand imbalances during $t-1$ (for example, if desired/planned demand exceeds realized supply, the price is raised). As such, prices are updated iteratively based on the degree to which expectations were met in the preceding production period. (We have left the mathematical language of the model relatively indifferent to several interpretations of the pricing mechanism that may be imposed upon it. The degree to which capitalists are able to influence the market is not strictly specified by the mathematics itself.)

The processes described here are iterated each time period through computational simulations, thereby investigating aggregate capital accumulation without any a priori presumption of equilibrium. What follows are explorations of the terrain that emerges from considering these principles in dynamic interaction, with each path departing from different initial distributions of capital, different prices, and different rates (expressed through changing parameter values) of the various social processes through which capitalists adjust their behavior in order to pursue profit in a dynamic environment with which their strategies are coevolving. Given the complexity of even this small part of the capitalist space economy, it is easy to focus on these dynamics alone while forgetting that they must be thought of as embedded in a variety of other processes which are excluded from the model. This incompleteness or openness must be acknowledged if we are to avoid determinism.

3 Simulations, (dis)equilibrium, and causality

Even the stylized, two-region, two-industry system described in section 2 is too rich to be exhaustively solved, given present-day mathematical knowledge (and certainly beyond the everyday calculations and inclinations of capitalists themselves). Thus, we used computer simulations to explore the dynamical tendencies of the system. The detailed parameter values and initial conditions underlying the computations for various figures are listed in appendix B (see http://dx.doi.org/10.1068/a411ap).

The high-dimensional nature of accumulation, even in this highly simplified case, poses difficulties for comprehensive visualization of all of the dynamics in two-dimensional graphics. Thus, in the figures and discussion to follow, we have chosen to compare graphs of one region's price ratio between the commodities, as one partial window into the changing states of much more complicated political economic trajectories. The parameters we used are such that both regions have the same production technologies, and transportation costs are negligible—assumptions which allow us to approximate a 'flat' world of extreme space–time compression and homogenization, so we can assess claims that geography ceases to matter in such a world.

3.1 Dynamical trajectories and emergent equilibrium

For the parameter values chosen for our numerical experiments, equilibrium-based mathematical reasoning provides a rationale for the assumption that the ratio of prices for the two commodities would be approximately 1.633 (calculated here with the values given in appendix B, following Sheppard and Barnes, 1990, pages 84–87). Such a ratio would, in these theories, allow markets to clear and the golden growth path to be obtained, given the physical technical requirements of production and the desire to maximize returns on the capital advanced. We include a horizontal line at this putative computed equilibrium price ratio for reference on some graphs, but it is important to recall that a concept of the golden growth path is not directly incorporated in the model itself.

According to our simulations, however, the political economic trajectories that result from iteratively applying the minimal set of classical political economic assumptions of social process described in section 2 often do reach some variation of the golden growth path, making the latter an emergent feature of the model. The clearest such trajectory displays 'convergence': a monotonic approach to the equilibrium price ratios and to balanced and equal growth and profit rates in all regions and sectors (figure 1).

For other parameters, model trajectories may dance around the golden growth path but never reside along it. The golden growth path has become locally unstable. Perpetual oscillations may occur, in which the spatiotemporal economy settles down into regular disequilibrium dynamics, with prices in one commodity oscillating around the warranted ratio and economic expansion coming through cycles of boom and bust, negatively correlated across regions (figure 2). A form of golden growth deterministic chaos is also possible, in which order and randomness are interwoven (figure 3). In golden growth chaos, noisy variation in the price ratio occasionally emerges, with periods of clearly resonating oscillation, albeit with the system still remaining close to the plotted golden growth price ratio (though these aforementioned runs have oscillatory price ratios whose averages taken over time appear to deviate by several percent

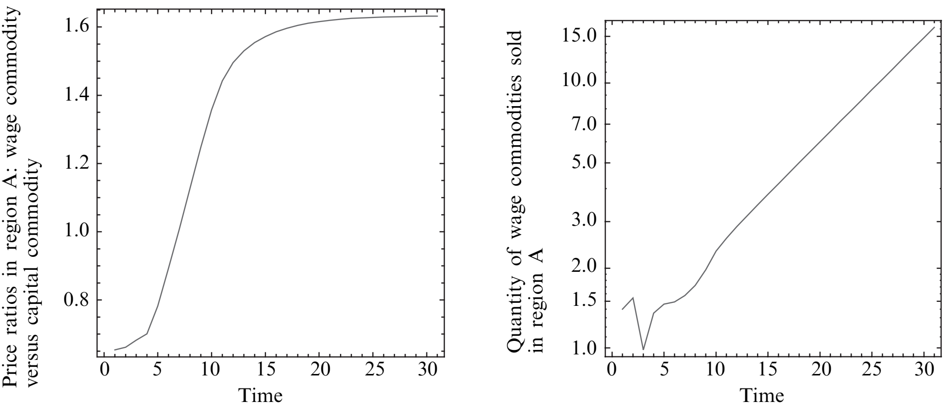

Figure 1. An example of convergence to the golden growth path.

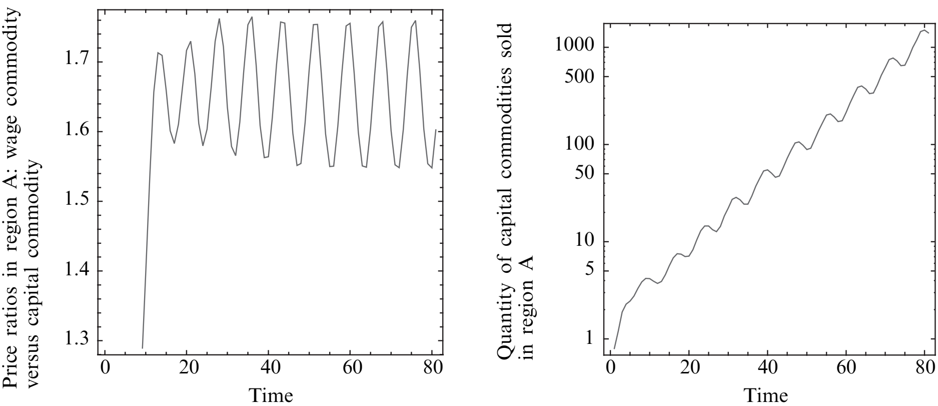

Figure 2. An example of oscillation around the golden growth path.

from the value warranted by equilibrium analysis). In these last two cases, the golden growth path of equilibrium theory has a spectral presence—eternally inaccessible, but always present!

This explicit model of relationally connected social processes generates a variety of outcomes that would not be immediately obvious when its principles are considered in isolation, and when optimization and equilibrium are assumed. Our actors have been placed in a dynamic context, in which they possess limited information and face significant uncertainties. Furthermore, we have specified the sociospatial processes as both taking time and lagged, which further opens up room for persistently far-from-equilibrium behaviors. In certain combinations of parameters, convergence may emerge. In other combinations various types of oscillation are conceivable, with differences in timings leading to complicated dances.

Yet these results do suggest a number of relatively convergent trajectories, or a certain degree of stability for our capitalism. This is consistent with the long-term persistence of capitalism notwithstanding its internal instabilities. Through fostering an emergent equilibrium guiding the dynamics—in this case, perhaps, by linking price and quantity spheres through more explicit accounting for money and in quantity—we have theorized a dynamic model capable of exploring plausible/realistic reactions to disturbance. Some results, however, take us considerably further from the apparent harmony of the golden growth path.

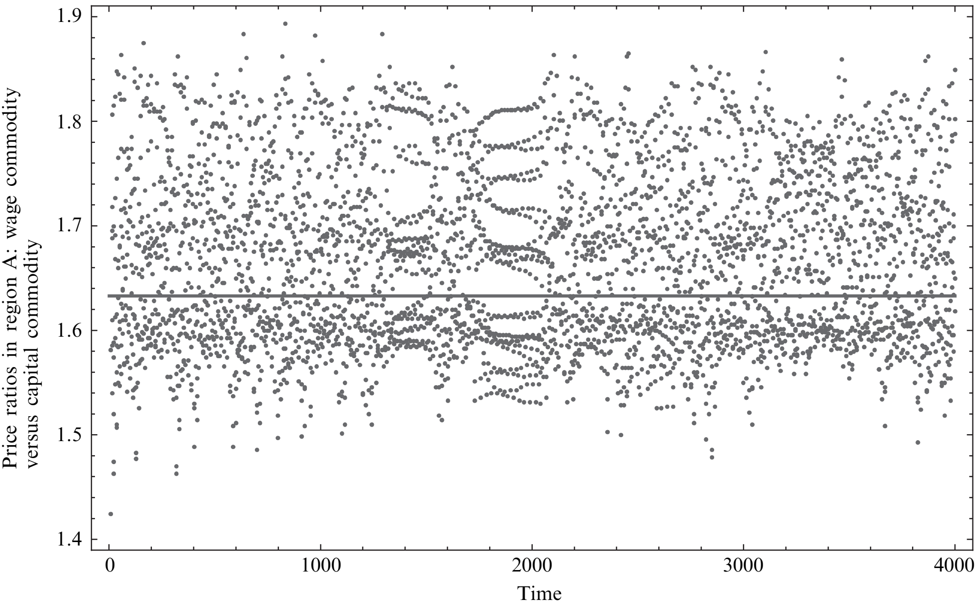

Figure 3. An example of gold growth chaos. This figure differs from previous figures in that a line has not been drawn through successive price ratios. Although this masks the fact that the behavior remains strongly cyclical, it reveals the intricate and shifting relationship between order and disorder in golden growth chaos. The horizontal line is drawn at the price ratio warranted by equilibrium theory.

3.2 Dynamical trajectories of collapse and polarization

For certain initial conditions and parameter values the model raises the specter of collapse in production, either across regions or within a subset of regions and sectors. For example, in some parameter regimes, a tipping point seems to be reached in spatial competition. At this point, what had been slowly growing oscillations in the regional shares of production within a certain sector suddenly diverge catastrophically, with all of the production in a sector ending up in just one region (figure 4). This is quite unlike the dynamics in figures 1 – 3, in which significant production of each sector persists within both regions.

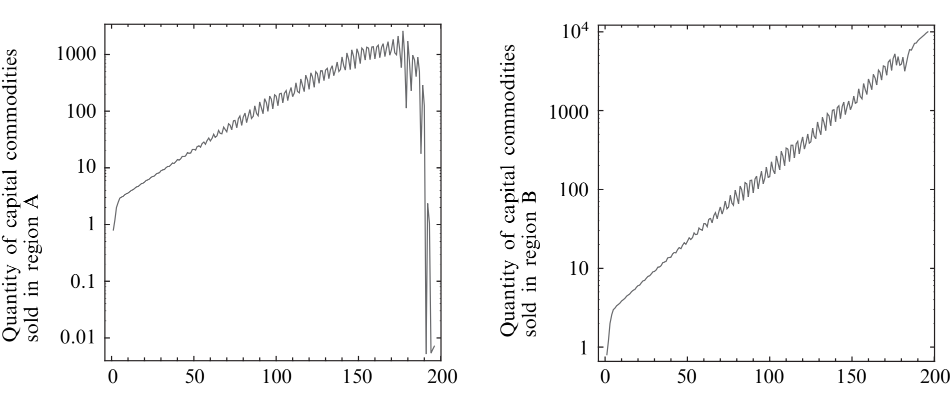

Figure 4. An example of interregional competition leading to collapse of the capital-goods sector in one region.

It has become a 'common sense' of our intellectual age that in a flat world everything should be in balance, as a level playing field equalizes opportunities for all (Friedman, 2005). In short, geography disappears. Regional specialization of production is an equilibrium outcome in trade theory, but only if production technologies, endowments, and/or accessibility vary between regions. In figure 4 collapse/specialization occurs when regions are identical in all respects, and represents a profound departure from equilibrium, as a result of dynamical interplay among interconnected regions. Since our use of a dynamical nonequilibrium model is the critical factor in enabling this particular polarization, it should be noted that the polarization occurs when the costs and barriers to quickly adjusting and profit-seeking interregional trade and investment, including transport costs, were quite low or were zero. In short, regional specialization is a dynamic outcome that occurs here because of the *lack* of trade barriers. From this nonequilibrium perspective, at least, geography matters even when the costs of distance do not.

The 'new' trade theory does generate equilibria with regional specialization when regions are identical (Krugman, 1980), but must invoke increasing returns for this to be possible. There are no increasing returns to scale in our model. Instead, we see evidence of an emergent causality at work here. We placed a collection of economic decision rules in dynamic relation with one another—price-conscious purchasing decisions, markets operating according to supply and demand, actors pursuing the greatest returns on their investments, among others. Individually, or even as parts of a carefully constructed equilibrium, these rules would seem to suggest some variation on a harmonious, spatially balanced, market outcome. However, in our case, a markedly different economic geography emerges. Dynamic geographies out of equilibrium can have different values for the long-term averages which (perhaps inspired by equilibrium thinking) might be used to imperfectly summarize their behavior, features which are not intuitively obvious from analysis oriented toward equilibrium.

3.3 The short and long run: unidirectional versus emergent causality

Many different trajectories are possible for a capitalist space economy in the event that there is a shift in the relative rates at which its various social processes occur. As these shifts in parameters are, in some sense, external to the model itself, we might view them as types of 'disturbances'. What happens after a disturbance?

Consider several seemingly similar trajectories, each chosen from the set of economic narratives that oscillate around the golden growth path of equilibrium theory (figure 5). Each simulation begins from the same initial disequilibrium condition in which the price ratio in region A is below the ratio 'warranted' by equilibrium theories (plotted as a dotted line). Nevertheless, each line then depicts one of many different trajectories that this price ratio might follow, given various speeds at which social processes of economic adjustment occur (implemented through changes in model parameters), or given various levels for the prices in the other region (appendix B). (This set of variations was chosen for the ease of display here—more 'extreme' parameter values may lead to larger oscillations or to divergent dynamics. These would change the details of any explanation, but would not alter the theoretical issues addressed below.)

These trajectories show quite different tendencies in the short, medium, and long run. Visually, the trajectories seem very similar at first, but they diverge over time in terms of the amplitude, shape, and timing of the oscillations. For the first two time steps, all is well. By this, we mean that conventional wisdom holds—all runs show that the low prices in this region/sector lead to more demand, in comparison with supply, for that region/sector relative to its competing region/sector, as expected. Prices then

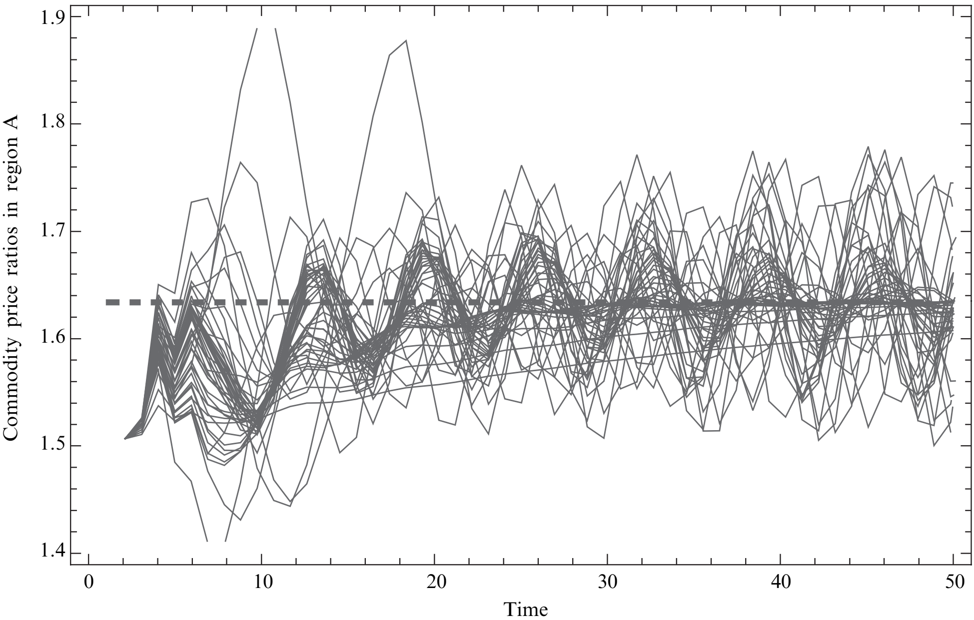

Figure 5. Beginning from the same price ratio in region A, many paths are shown differing as a result of having either different parameter values or shifts in other initial conditions.

rise in that sector/region. For the first few production periods, the dominant feedback mechanism is what might be said to be the economically obvious one: the logic of the pricing equation. Straightforward unidirectional causality suffices to account for the dynamics.

Yet the situation has become much messier by period 5: by then, secondary and tertiary feedbacks have taken over. A variety of behaviors are observed between periods 5 and 10. Some paths overshoot the warranted price ratio: perhaps low prices led to a rate of return that was undesirably low in a sector/region, and even as its revenue and the likelihood of being a supplier rose, its investment capital was being reinvested in opportunities that brought higher rates of return, thus letting local supply grow more slowly than demand? For a larger number of trajectories in the figure, the price ratio seems to stagger around, but stays below the warranted rate for some time. Thus, more generally, could it be that imbalances in regional production and pricing have triggered capital flows between sectors/regions as investors seek the best rates of return, only to disturb the quantity sphere and set off new disturbances in the price sphere?

Our question marks indicate that such interpretations are merely possibilities. Each path would need to be examined in much greater detail, with reference made to trajectories of the many other variables not plotted here, before they can be substantiated. But that is not our purpose in this paper. Here, we highlight the ways in which the dynamics change in these various families of runs over time in order to make a more abstract point about causality in nonlinear dynamic space economies in general.

In the short run, after a ‘disturbance’ to the system of some sort, the spatiotemporal dynamics of the space economy are dominated by unidirectional cause and effect. The processes of economic adjustment that are most proximate (measured not by geographical distance but in terms of the network of mathematical equations, which have temporal and spatial elements) to the disturbance dominate. In the medium run, the disturbance has rippled through the system of equations far beyond the original disturbance in the pricing sphere. Unintended consequences are ubiquitous, and a variety of potentially unexpected dynamics can occur, depending on the configurations and strengths of the linkages between variables. It becomes difficult to understand these dynamics in terms of any single law because unidirectional causality gradually becomes dominated by emergent causality. Economic change cannot be understood in terms of any single theoretical principle, but through a complicated, nonlinear interaction among theoretical principles.

In the longer run, paths tend to reach one of the classes of several abstract behaviors described in the previous sections (figures 1 – 4). All individuality of the social processes involved has been subsumed within the emergent result of the interaction between all regions and sectors. The economic variables in our model, its regions, its various moments in time are dialectically interrelated in our method of mathematical theorization. Simulations can provide a language that avoids both violent reductionism and naïve holism to explore how everything is indeed overdetermined.

From an equilibrium-oriented and closed-system standpoint, such differences between short and long run may well seem trivial; such information is not considered particularly meaningful. It suffices to know whether the economy returns to an equilibrium path of growth through a classification of long-run fates. Speed of adjustment after a recession is of great concern to those affected by it, but the relative rapidity of convergence usually does not make a difference to the classification of particular trajectories. Once we recall the fact that all such systems are actually incomplete and open, however, shorter run transient paths may become of much greater analytical interest.

3.4 From the eternal to the ephemeral: embracing incomplete systems

"'We do not record them', said the geographer, 'because they are ephemeral'."

Saint Exupéry's *The Little Prince*

When a system can convincingly be conceived of as composed of unproblematic objects whose underlying laws are understood, and as not subject to perturbations from without (or, that those perturbations are at least well characterized), then it can be useful to analyze its possible long-term tendencies. Yet the 'economy' is not an isolated system, operating according to iron laws divorced from society and the larger environment. Generating 'an economy' through simple mathematical equations, therefore, is both too simplistic and too limited for us to expect that many of the long-run fates found on the computer will ever be realized. Economic models are always too partial, and our worlds too overflowing. Yet the 'blooming, buzzing confusion' of the world need not greatly diminish the worth of thinking through models. We need, however, a framework in which mathematical theories might begin to communicate productively with much of the richness that inevitably lies beyond them. Recognizing that much of this richness is often elegantly called forth by qualitative discourse is important. Yet incorporating such richness in conversations with theoretical quantitative discourse has often been difficult. If we want to theorize a world of flux, and make the partiality of quantitative narratives a strength rather than a weakness in making sense of it, we need to develop methods capable of deftly considering everything from the short to the long term.

Suppose, as we have, that many of the principles we wrote in mathematical language operate, more or less, in a capitalist space economy. We can gain useful insights from placing them in relation with one another using computer simulations to explore the many possible trajectories (of which an economy can occupy only one at a time), as we have done above. Recognizing that an 'economy' is an incomplete and rather unruly discursive construction, however, means acknowledging that it will often be subject to a variety of disturbances: the values of its variables, or its parameters, can suddenly shift due to factors we have not explicitly theorized within the mathematical characterization. This limits the value of any single trajectory generated by the model, because continual disturbances bring about continual reconfigurations of the trajectories (as well as the phase diagram) and of the position of a given economy within it.

Any mathematical model of a space economy is incomplete, a distorted 'shadow' of certain parts of the higher dimensional 'phase space' embracing it. Thus we must acknowledge and debate how to incorporate the qualitative dimensions of social context within the multidimensional social phase space in order to avoid fetishizing the mathematical solutions to the lower dimensional and incomplete model as closed and teleological. To 'close' mathematical systems and compute their solutions through endless repeated iteration, as natural as such a procedure may be to many modelers today, is too simple. If we conceptualize the capitalist space economy as merely one part of a larger socionatural system, one moment of a dialectically interrelated larger world, we cannot limit ourselves to questions about the long-term fate of our equations. Appreciating the qualitative (and untheorized quantitative) dimensions suggests that one might create quantitative economic geographic narratives by 'stringing together' segments of many different short-run mathematical model paths.

At turning points in the narrative, within the world of the mathematical model, we might observe characteristic patterns of 'disturbance', shadows projected from higher 'qualitative' dimensions. Through the process of stepping back from fetishizing individual mathematical model trajectories as real, but instead stitching them together according to a qualitative script, can one not begin to imagine a method of multilingual theorization that draws strengths both from quantitative and from qualitative languages?

3.5 Critical considerations in delimiting the limitless

Of course, such a methodological path cannot be casually traveled. On the one hand, one needs methodologies for acknowledging, and conversing productively with, elements explicitly beyond the mathematical formulation, and the disposition to do so. On the other hand, we argue that there is unexpected theoretical virtue in placing principles expressed in a mathematical system in relation with one another. If so, it is unavoidable that connections will be missed between those processes that are directly in the mathematically theorized system and those other (perhaps qualitatively narrated) processes which remain the concern of the more expansive, 'open' mathematical method being developed here. If we are conscious of these tensions, however, we can work thoughtfully to minimize their effects.

Consciously choosing the social boundaries of the mathematics becomes particularly important, if always fraught. Even the dynamical variables apparently included 'within' a model themselves aggregate large numbers of actors and phenomena—hidden elements that always resist any such aggregation into unproblematic variables through their actions, to a greater or lesser degree. Thus there is always 'noise' internal to any variable and model, regardless of choice of 'boundary'. Upon establishing a model boundary, there is also external 'noise.' Thus in the model developed here, we chose not to directly mathematically express trajectories of technical change, changes in distribution of the economic product between capital and labor, or changes in any number of the parameters of the model, among others. Yet we require a methodology capable of meaningfully contemplating the implications of such continual 'disruptions.' It is likely that there will be crucial interactions between our model variables/dimensions and nonmodeled variables/dimensions—interactions that make modeling within a narrow set of variables inadequate, even in the presence of discrete occasional perturbations to model variables in the form of these 'disturbances'. In other words, the 'texture' of the phase space we are examining is likely to change considerably over time through its dialectical linkage to things outside of our model. The challenge is to choose system boundaries while being relatively conscious of the costs and benefits of any particular choice, while retaining a constructive, respectful modesty toward that which we cannot incorporate.

4 Conclusion

We have explored how the use of certain mathematics may be productively interpreted in the service of postpositivist narrations of an open-ended, yet still intricately structured, world of flux. Such approaches have affinities with—and the potential to contribute to—qualitative theoretical work currently popular in sociospatial theory, inspired by such philosophers as Marx and Deleuze. We do not agree with those complexity theorists who claim that mathematical approaches can replace such qualitative insights with some new all-embracing nonlinear general systems theory of the world: qualitative approaches remain important. Yet the two kinds of approaches can complement—and, to an undetermined extent, integrate with—one another. For all their limitations, mathematics and computation force theorists to be clear about the simplifying assumptions made in constructing theories, models, and abstractions. They also allow us to trace precisely the implications of the assumptions made—though this is far from trivial. Yet mathematical modelers should never lose sight of the particular limitations associated with this particular language and should engage with the insights of qualitative theory.

The model in this paper—a nonteleological model of the complex relational spatial dynamics of production, circulation, and accumulation in a spatially extensive capitalist economy—is used to demonstrate many of our claims. Equilibria may emerge, but not because they are built into the theory from the outset. More commonly, they haunt (rather than determine) the dynamics of capital accumulation via regular and irregular cycles of uneven development; systemic deviation from equilibrium (extreme uneven development with local collapse and polarization) is also possible. The straightforward logics governing short-run dynamics quickly become dominated by much more complex and difficult to understand (or predict) dynamics of the long run, as complex emergent causal relations take over. Yet, recognizing the necessary incompleteness of our model also reminds us that the short run may matter more—the teleology of individual model runs can be tempered by stringing together short-run dynamical paths. Redefining modeling to incorporate such practices would allow our narratives to respond to the higher dimensional, often qualitative, context within which any model is embedded (and which it may shape).

The most serious problem with mathematics is not its inherent limitations: all speech is productive and limited, whether one speaks the theoretical English of the contemporary American humanities or that of differential equations. Rather, the problem lies with the way in which we conventionally use and interpret mathematics: we are too accustomed to using models in a teleological, deterministic, and closed manner. If we build our equilibrium outcomes into our models from the outset, we should not be surprised when they reproduce a narrow equilibrium thinking. If we make the convenient, but ultimately deafening, assumption that our system of equations is, in some critical sense, 'complete', we can be tempted to be too literal and simplistic in interpreting the solutions to our equations as deterministic and teleological narratives. This also undermines collaboration between qualitative and quantitative discourses.

We do need more conversations about how models should be set up and how their results become possible. But we can aspire to something more than a respect for a reified difference, a coexistence of eclectic methods juxtaposed on paper but not in motion. Let us develop approaches that hybridize the qualitative and quantitative! Better, let us not partake of the social processes that make this dichotomy meaningful in the first place.

We have sought to recognize the incompleteness of any model, and to approach this feature as a virtue rather than as a vice. If we take too literally closed mathematical narratives—narratives which arise from the endless repetition, without difference, of mathematically expressed social relations—we too narrowly delimit the role of human agency in our thinking. In the example studied here, that of theorizing societies in which capital circulates, we have much to gain by continuing to develop methods to embrace mathematical expressions of social relations in their incompleteness. Among other benefits, we might well be able to contribute to the explication and the extension of what Thoburn (2003) locates as Deleuze's (1994 [1968]) seeming endorsement in *Difference and Repetition* of an Althusserian theory of capital, which is "premised on processes of difference and variation rather than contradiction" (Thoburn, 2003, page 153, highlighting Deleuze, 1994 [1968], page 207).

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